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Bluebell Primary School

Calculations Policy:-

Guidance, examples and full progression.

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Aims and Rationale

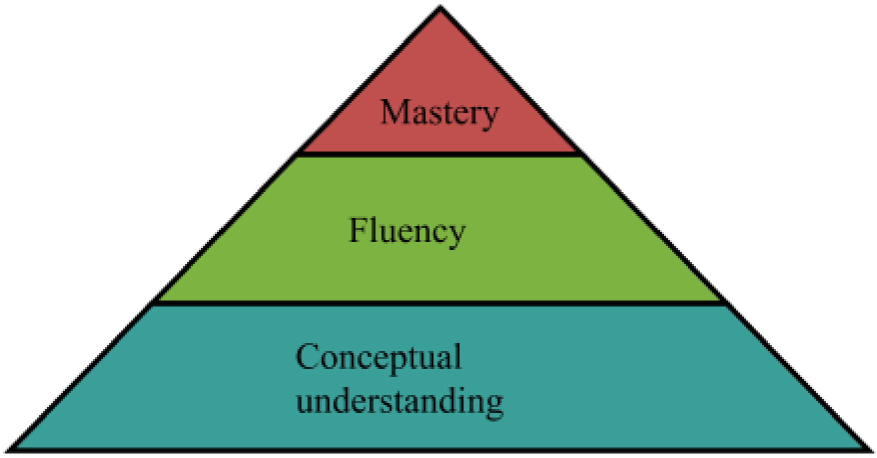
Aims of the policy.

This policy is designed to create a common way of teaching calculation strategies at Bluebell Primary Academy, and to provide detailed guidance and information to staff to enable them to effectively support the development of children's calculation skills.

Rationale behind the policy.

The effective teaching of **mental calculations** has consistently been found to underpin success in written calculations and therefore mental calculations forms a **central part of this policy.**

Research[1] consistently states that the most effective calculation strategies are those which are built on **conceptual understanding-** which foster a real understanding of the structure of mathematics and why the calculation methods work. Conceptual understanding leads to fluency which in turn leads to mastery.



Therefore, the calculation methods set out in this policy are those which are clearly based on a conceptual understanding of maths and which also allow children to experience the **joy of discovery.** The methods are **relational** *(encourage children to see the relationships, make connections and see why something works)* rather than **instructional** *(relies on children simply following a set of instructions with no understanding).*

The formal algorithms for calculation are introduced **conceptually** once children have a secure **conceptual understanding** of the operation and are becoming fluent. Formal algorithms are to be introduced in the way outlined in this policy to ensure the link between the conceptual understanding and the algorithm is maintained. Children should not simply be taught to follow a set of instructions without understanding. <u>Children should not be forced onto the advanced stages of formal algorithms until they have a secure conceptual understanding.</u>

The policy is also based on the principles of math's teaching in the far east, and develops a small number of concepts and methods throughout the children's mathematics career, rather than switching and changing between concepts. This allows for conceptual understanding, fluency and therefore mastery to be developed at an earlier stage in children's mathematics career.

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[1]E.g.- the Norfolk Calculations project- amongst many others. The latest Primary Norfolk Calculations research can be found using the following reference:- Borthwick, A. and Harcourt-Heath,M (2012) Calculating: what can year 5 children do now? Proceedings of the British Society for Research into Learning Mathematics' 32 (3) pp. 25-30

Structure of the policy.

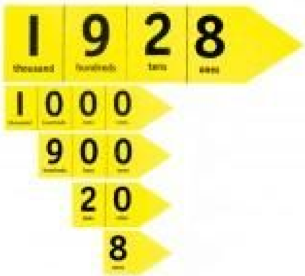
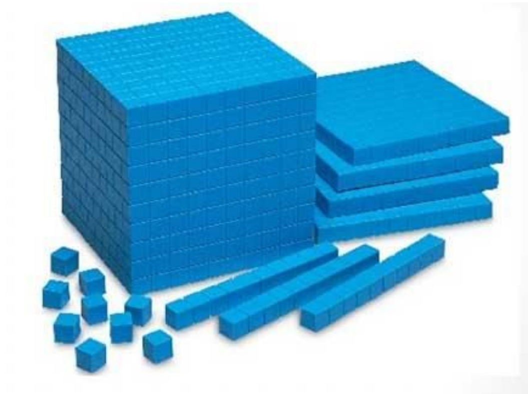
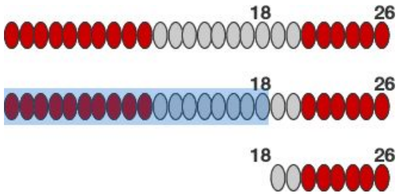
For each operation the following is provided:-

- An **overview** of the operation in a nutshell.
- **Key Vocabulary** related to the operation.
- **Early Learning** - how children first experience and begin to develop knowledge and understanding of the operations. In EYFS this should be supplemented by approaches from Numicon:Firm Foundations.
- **Mental Strategies** - key strategies and expectations for mental calculations related to each objective.
- **Written Strategies** - for each operation at least two different written strategies are outlined. These strategies are outlined as a progression, and there is no fixed expectations per year group- it is important that children work at the level which supports and develops their conceptually understanding and fluency.

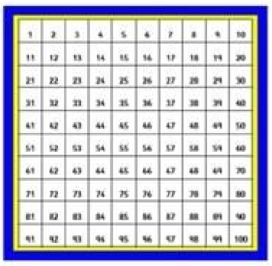
Representations

Representations are vitally important throughout a child's maths education. Representations provide a 'hook' for children to 'hang' mathematical concepts, and allow children to manipulate and later visualise the structure of mathematics.
Representations are therefore a significant aid in developing conceptual understanding. Different concepts can be represented using the same resource/representation depending on the child's age and stage of mathematical development.

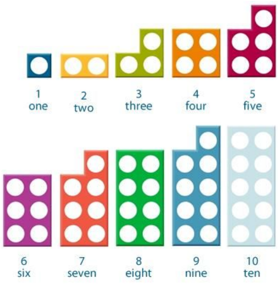
Below are the key representations, presented in alphabetical order, that will be in use throughout a child's maths education at Bluebell.

Arrow Cards  	Base 10 apparatus/Dienes 
Bead Strings  	Counters 
Counting Sticks  	Cubes 
Cuisenaire Rods 	Number Lines (+ blank number lines) 
Multiplication Grids	Hundred Squares

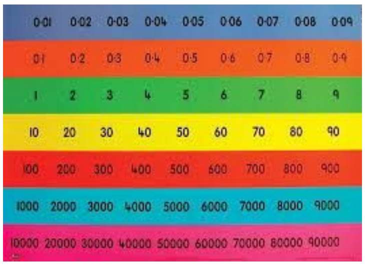
×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100



Numicon

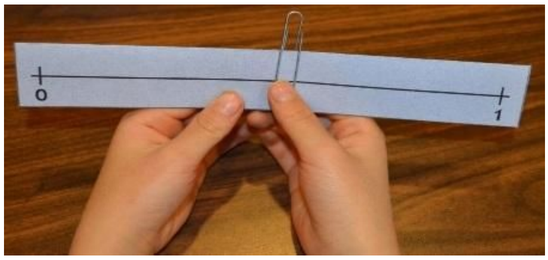


Place Value Chart (Gattegno charts)



Place value counters

Thinking Strips/The bar model



Fundamental concepts which underpin this policy.

Throughout the policy, key notes are highlighted in red boxes. These are around the key concepts that children need to be secure with in order to calculate fluently and therefore begin to show mastery.

The following fundamental concepts underpin every operation within the policy and are fundamental to the teaching of calculations in our school.

Fundamental concept 1- Place value.
The methods in this policy are based on a **place value** system of number, as opposed to a digit value system. This means that when referring to different digits in a number, we refer to them as the place value of the digit, rather than simply the digit name. For example, in the number 7321 we would **always** refer to the digits as being 'seven thousand, three hundred, twenty and one' rather than 'seven, three, two, one'.

For the avoidance of doubt, under the new curriculum, the term 'ones' is expected to be used in place of 'units.

Fundamental concepts 2 and 3- Base 10 and exchange.
Children need to understand that our number system is a 'base 10' system which relies on exchanges to work. Place value representations, such as Base 10 (Dienes) and place value counters are essential in supporting the developing of these concepts. Place value mats/grids can also help when children are manipulating these representations.

This means that children need to understand that you can exchange **10** 'ones' for **1** 'ten', **10** 'tens' for **1** hundred etc…

They need to understand that when giving their final answer, these exchanges **must** have occurred (e.g. you can't record the number 'one hundred and eleven' as 1011 as the eleven 1's would have to be exchanged for one '10')

Children also need to understand that exchange *can* ***temporality*** happen in either direction if this assist with a calculation. E.g. 456 can be *temporality* thought of as ' four '100's', four '10's' and sixteen '1's'.

Fundamental concept 4- Efficiency.
Children need to be encouraged to work efficiently. Efficient working means a method of working that is **accurate** and **quick.** Speed or accuracy alone does not create an efficient method.

The efficient methods children use will develop with their conceptual understanding. For example, for a particular child, an informal written method may, at some points in their mathematics education, or for some calculation types, be more efficient than the formal written method as the formal written method is leading to mistakes (which would point to a lack of conceptual understanding which needs to be developed though the informal written methods).

Fundamental concept 5- Meaning of the equals sign.

It is important that children understand that the equals symbol (=) means 'balance' rather than 'the answer'.

When presenting problems, always make sure they are presented balanced- i.e. with a ? or other symbol after the = symbol- for example- 6+2=?, 67+4=[] c=321+345

Presenting problems in a range of formats (e.g. 6 + ?=10, 10+6=?, ?+4=10) also helps re-enforce this definition..

Fundamental concept 6- Accurate vocabulary.

Children should be able to talk about their work using accurate and concise mathematical vocabulary, and this needs to be modeled by adults.

Note the words below, which are often used incorrectly- and their correct definitions

Sum- applies to the answer to an addition question only. You are not carrying out multiplication 'sums', rather multiplication 'questions' or 'problems'.

Negative – should be used to describe numbers below 0- not minus.

Additive Reasoning

Progression in Calculations

(Addition and Subtraction)

Additive Reasoning- Addition

Addition involves the adding together of sets of objects, or the adding of more objects to a set. Addition is commutative.   The inverse of addition is subtraction.

Key Vocabulary
add, altogether, sum, total, increase, more

Early Learning- Addition

Children will relate addition to the combining of 2 groups:

For example: 3 + 2 = 5

Count out 3,  count out 2. 

Put together and count = 5

They develop ways of recording calculations using pictures, etc. Children are also encouraged to make use of fingers - these are a constantly available resource for calculations!

Key Concepts
Commutative law of addition. Numbers can be added in any order. For example, 7 + 12 = 12 + 7 The same applies when more than one number is added

Mental Strategies- Addition

Counting forwards and backwards

Children first meet counting by beginning at one and counting on in ones. Their sense of number is extended by beginning at different numbers and counting forwards and backwards in steps, not only of ones, but also of twos, fives, tens, hundreds, tenths and so on.

The image of a number line helps them to appreciate the idea of counting forwards and backwards. They will also learn that, when they add two numbers together, it is generally easier to count on from the larger number rather than the smaller. You will need to review children's 'counting on' strategies, then show them and encourage them to adopt more efficient methods.

Progressio n	4+5=? by counting on in one from 4. 10+6=? by counting in ones from 10 (or use place value knowledge). 23+5=? by counting in ones from 23.
	27+60=? by counting on in tens from 27. 50+38=? by counting on in tens then ones from 50. 35+15=? by counting on in steps of 5 from 35.
	47+58=? by counting on 50 from 47, then 3 to 100 then to 105. 570+300=? By counting on in hundreds from 570.
	3.2+0.6=? by counting on in tenths. 1.7+0.55=? by counting on in tenths and hundreths.

Partitioning

It is important that children are aware that numbers can be partitioned- both along the place value boundaries (canonically) and in other ways (non-canonically)

Progr ession	30+47=? by 30+40+7 23+45=? by 40+5 +20 +3 or 40+23+5
	55+37=? by 55+30+7 or 50+37+5
	42+28+51=? by 40+2+20+8+50+1 =40+20+50+8 +1 or by 42+20+50+8+1
	5.6+3.7=? by 5.6+3+0.7 540+380= ? by 540+300+80 or 540+360+20

Partitioning- Bridging Through Multiples of 10

An important aspect of having an appreciation of number is to know how close a number is to the next or the previous multiple of 10. For example, that 47 is 3 away from 50, or that 47 is 7 away from 40.

In mental addition, it is often useful to count on or back in two steps, bridging a multiple of 10. The empty number line, with multiples of 10 as 'landmarks', is helpful, since children can visualise jumping to them. For example, 6 + 7 is worked out in two jumps, first to 10, then to 13. The answer is the last point marked on the line, 13.

Progression	5+8 =? by 5+5+3 65+7=? by 65+5+2
	49+32=? by 49+1+31 57+34=? by 57+3+31
	1.4+1.7=? by 1.4+0.6+1.1 0.8+0.35=? by 0.8+0.2+0.15

Partitioning- Compensating

This strategy is useful for adding numbers that are close to a multiple of 10, such as numbers that end in 1 or 2, or 8 or 9. The number to be added is rounded to a multiple of 10 plus or minus a small number. For example, adding 9 is carried out by adding 10, then subtracting 1.
A similar strategy works for adding decimals that are close to whole numbers. For example:
1.4 + 2.9 = 1.4 + 3 – 0.1

Progression	34+9=? by 34+10-1 34+11=? by 34+10+1
	53+12 =? by 53+10+2 53+18 =? by 53+20-2 38+68=? by 38+70-2
	138+69=? by 138+70-1
	2 ½ + 1 ¾ by 2½ + 2 – ¼ 5.7+3.9 by 5.7+4.0-0.1

Partitioning- Using near doubles

If children have instant recall of doubles, they can use this information when adding two numbers that are very close to each other. So, knowing that 6 + 6 = 12, they can be encouraged to use this to help them find 7 + 6, rather than use a counting on strategy or bridging through 10.

Progression	6+7 is double 6 add 1 *or* double 7 subtract 1
	13+14 is double 13 add 1 *or* double 14 subtract 1 39+40 is double 40 and subtract 1
	18+16 is double 18 and subtract 2 *or* double 16 and add 2
	60+70 is double 60 and add 10 *or* double 70 and subtract 10. 75+76 is double 76 and subtract 1 *or* double 75 and add 1 160 + 170 is double 150 then add 10 then add 20 *or* double 160 and add 10 or double 170 and subtract 10 2.5+2.6 is double 2.5 add 0.1 or double 2.6 subtract 0.1

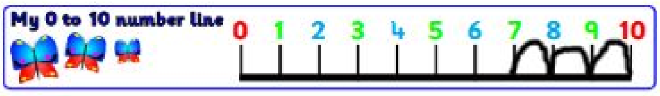
Written strategies for addition.

Key Concepts
Partitioning This is a fundamental concept for addition. Numbers can be **canonically** or **non-canonically.** **Canonical** Partitioning is 'splitting' a number into hundreds, tens, ones, etc… for example: **534** partitioned canonically **500 + 30 + 4** **3.456** partitioned canonically **3 + 0.4 + 0.05 + 0.006** **Non-canonical** Partitioning in other ways. For example: **534** partitioned non-canonically **250 + 250 + 34** OR **334 + 200**

STAGE A:- Counting up, leading to partitioning- number lines.

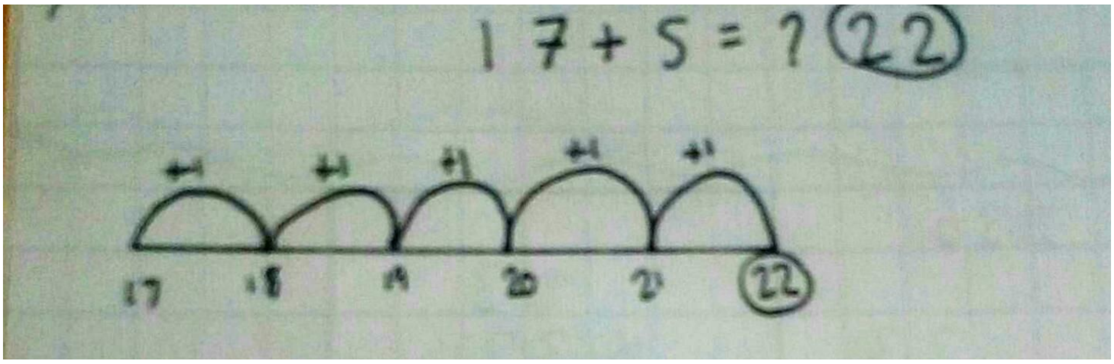
Children first complete addition by counting on printed number lines.

Step 1:- eg 7+3=?

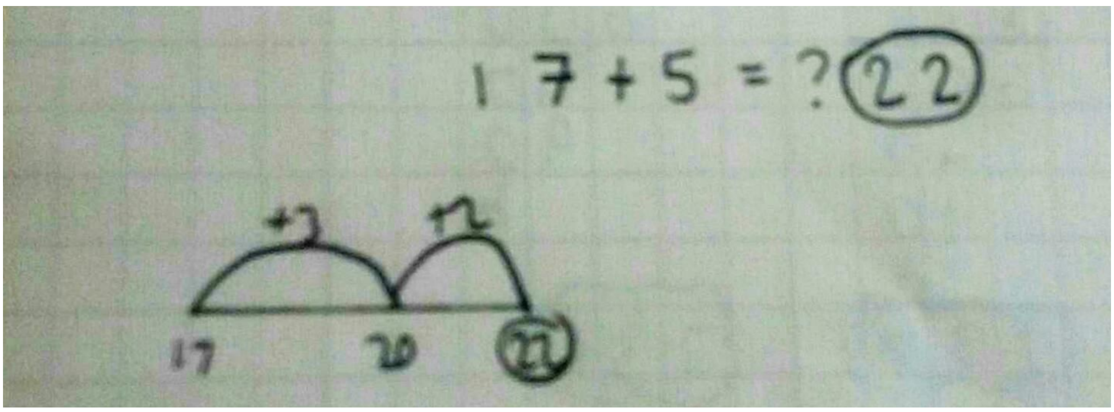


And begin to use blank number lines and extend this to numbers which bridge through 10.

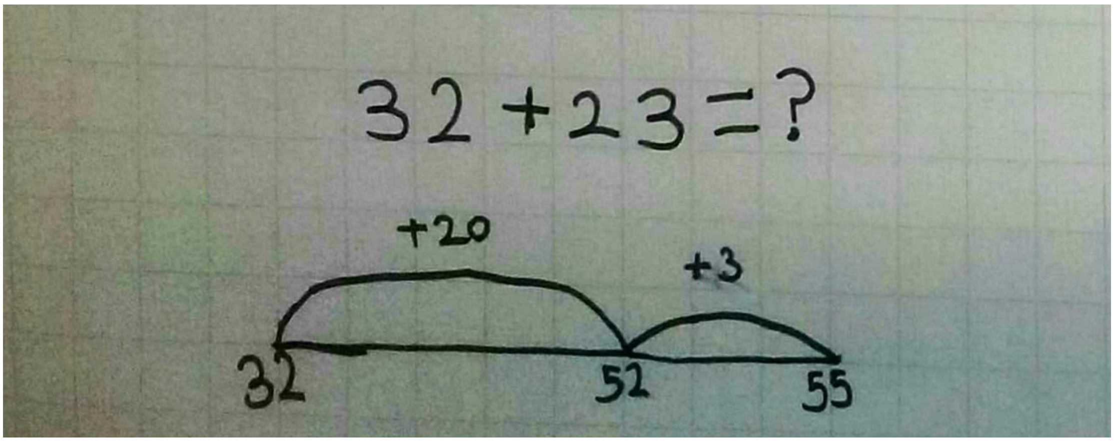
Step 2: 17 + 5=?



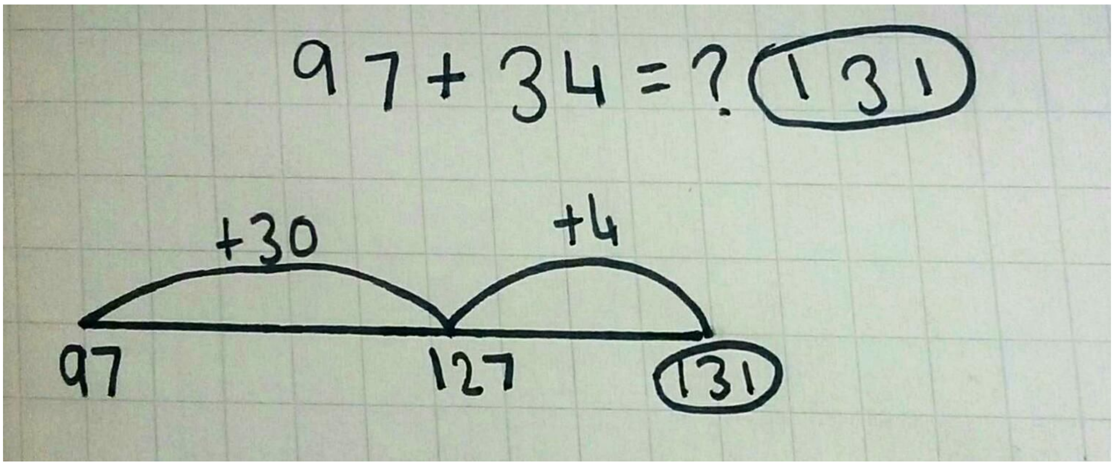
Children begin to jump in steps greater than 1, often adding to the next multiple of 10. As children become more confident they should be encouraged to use their **partitioning** skills to add the tens numbers first (larger steps)



Example: 32 + 23=?



Example: 97+34=?



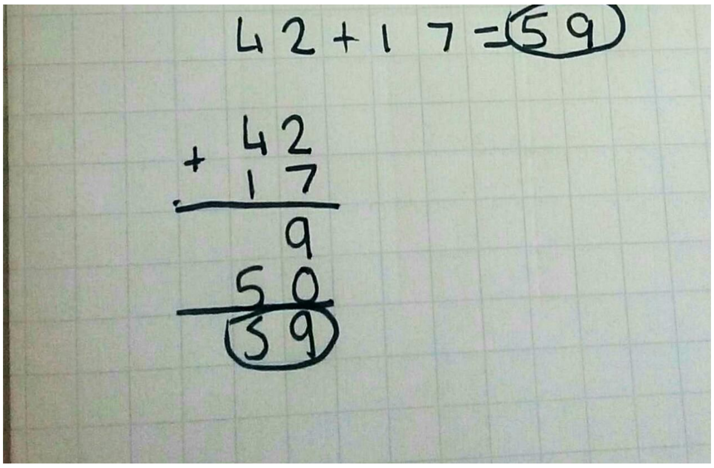
Support Step
When bridging through 10 and 100 children may find it easier to use **non-canonical partitioning** skills, before progressing to partitioning canonically e.g. 97 + 34 =

STAGE B:- Formal written method- partitioning.

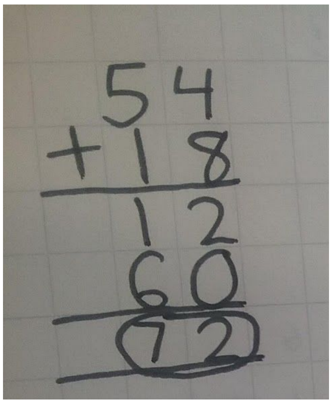
Children will use their **canonical partitioning** skills to record their calculations in a formal vertical method.

Initially children will use an expanded version of the method. Always adding the **lowest value** place first (e.g. ones, tenths etc)

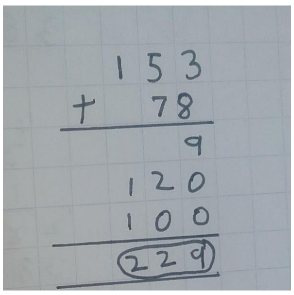
Example 1: 42 + 17=?



Example 2: 54+18=?

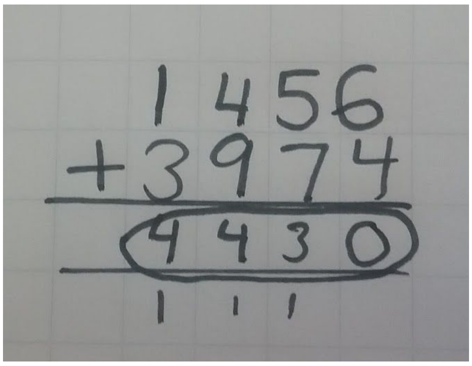


Example 3: eg 153+76=?

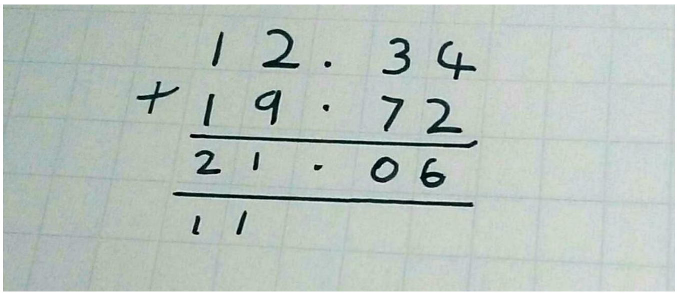


Once children show a secure conceptual understanding, they should begin to compact the method. The language of 'exchange' should be used- they exchange 10 ones for 1 ten, 10 tens for 1 hundred etc

Example 4: eg 1456+3974=?



Example 5: eg 12.34+ 19.72=?



Support Step
If children are struggling with the shorthand notation for the exchanges, the exchange can be recorded with its full value.

Additive Reasoning- <u>Subtraction</u>

Subtraction is the process of taking one quantity away from another, or finding the difference between two quantities. It is the inverse of addition and is not commutative, i.e. 5 - 3 is not the same as 3 - 5.

Early Learning- Subtraction

Key Vocabulary
subtract, take away, more than, less than, minus, find the difference, how many left, decrease, fewer than, difference between

In the early stages, children will be taught to **'take away'** one or two objects and find the new total.

For example: 5-2=?

5 take away 2 is 3

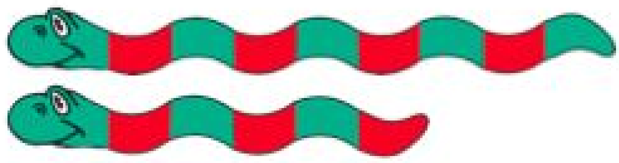
They develop ways of recording calculations using pictures etc.
5-2=3

Children will be taught to use the vocabulary involved in subtraction through practical activities and discussion e.g. take away, leave, how many left?, how many more to make?;

Children will also jump backwards on different sized number lines and tracks; physically taking away objects from a given set (throwing away/gone).

They will be taught to recognise differences in quantity in everyday objects and to find one less in practical contexts that relate to the children's experiences using various resources. From the very first stages, the children will be introduced to number lines including those with negative numbers and encouraged to visualise the calculation.

Early learning examples:

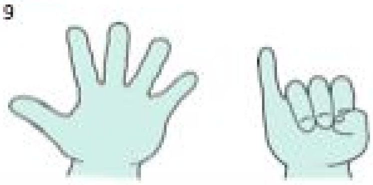


The difference is?

1 **less** than 10 is 9

10 **subtract** 1 equals 9

10 - 1 = 9



6 + ? = 10 ? + 6 = 10

10 – 6 = ? 10 – 4 = 6

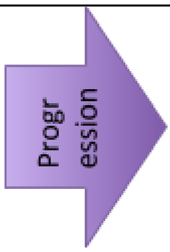
5 - ■ = 3 ■ - 2 = 3

Key models and images - Subtraction

Mental Strategies- Subtraction

Counting On or Back

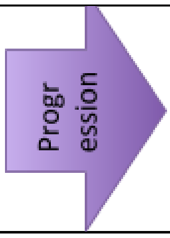
Children first meet counting back by beginning at one and counting back in ones. Their sense of number is extended by beginning at different numbers and counting forwards and backwards in steps, not only of ones, but also of twos, fives, tens, hundreds, tenths and so on. The image of a number line helps them to appreciate the idea of counting forwards and backwards.



Count on or back in ones, twos or tens.
Count on or back in tens and ones.
Count on or back in hundreds, tens and ones.
Count on or back in hundreds, tens, ones and tenths.
Count on or back in hundreds, tens, ones, tenths and hundredths.

Partitioning- Bridging

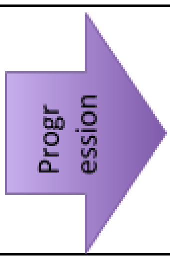
It is important for children to know that numbers can be partitioned into, for example, hundreds, tens and ones, so that 326 = 300 + 20 + 6. An important aspect of having an appreciation of number is to know how close a number is to the next or the previous multiple of 10: to recognise, for example, that 47 is 3 away from 50, or that 47 is 7 away from 40. In mental addition or subtraction, it is often useful to count on or back in two steps, bridging a multiple of 10.



Bridge through 10 and multiples of 10.
Bridge through multiples of 100 and 10.
Bridge through multiples of 1000, 100 and 10.
Bridge through whole numbers.

Partitioning- Compensating

This strategy is useful for subtracting numbers that are close to a multiple of 10, such as numbers that end in 1 or 2, or 8 or 9. The number to be subtracted is rounded to a multiple of 10 plus or minus a small number. For example, subtracting 18 is carried out by subtracting 20, then adding 2.
A similar strategy works for adding or subtracting decimals that are close to whole numbers.



Subtract 10 and adjust by 1.
Subtract a multiple of 10 and adjust by 1.
Subtract 10 or 20 and adjust.
Subtract a multiple of 10 and adjust.
Subtract a multiple of 10 or 100 and adjust.
Subtract a whole number and adjust.

Place Value and Related Calculations

Using their knowledge of place value, children can use facts they know to derive others.

Progression	Use knowledge of place value and related calculations e.g. work out 180-120 =60 using 18-12=6.
	Use knowledge of place value and related calculations e.g. work out 6.3–4.8=? using 63–48.
	Use knowledge of place value and related calculations e.g. 680-430, 6.8-4.3, 0.68-0.43 can all be worked out using the related calculation 68–43.

Partitioning Bridging Through 60 to Calculate a Time Interval

Time is a universal non-metric measure. A digital clock displaying 9.59 will, in two minutes time, read 10.01 not 9.61. When children use minutes and hours to calculate time intervals, they have to bridge through 60. So to find the time 20 minutes after 8.50am, for example, children might say 8.50am plus 10 minutes takes us to 9.00am, then add another 10 minutes.

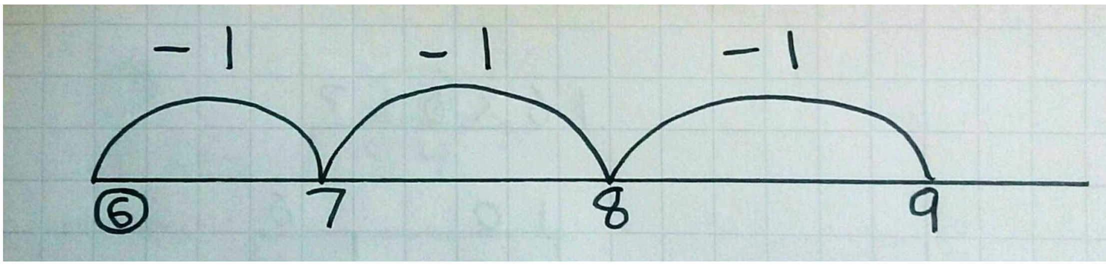
Progression	Partition: count on or back in minutes and hours, bridging through 60 (analogue times).
	Partition: count on or back in minutes and hours, bridging through 60 (analogue and digital times).
	Partition: count on or back in minutes and hours, bridging through 60 (analogue and digital times, 12-hour and 24-hour clock).

Key Concepts
It is important that children understand that subtraction can be viewed as take away as well as finding the difference. For example: 12 - 3 can also be described as 'What is the difference between 12 and 3? **Subtract** With the above in mind, it is therefore important that children understand that the '-' symbol refers to subtract rather than take-away when verbalizing their maths. **Partitioning** It is important that children understand the how numbers partition, especially when moving towards the formal written method. Commutativity Children should know that subtraction is not commutative. For instance 5 - 3 is not the same as 3 - 5.

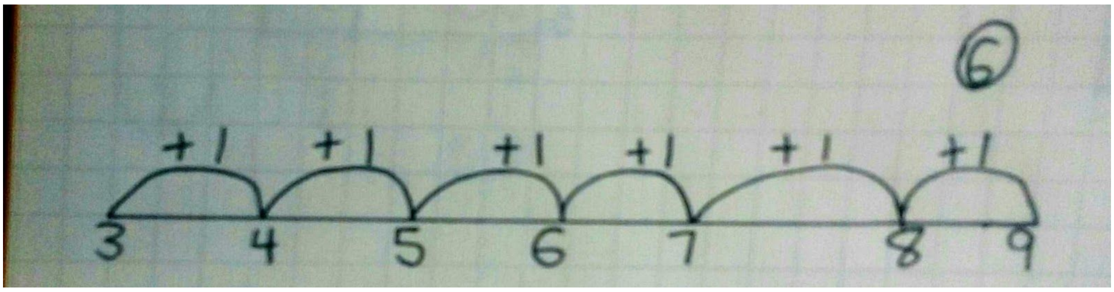
Written Methods- Subtraction

STAGE A:- Informal written method- Number line.

Example 1: eg 19-5=?
Taking away:-

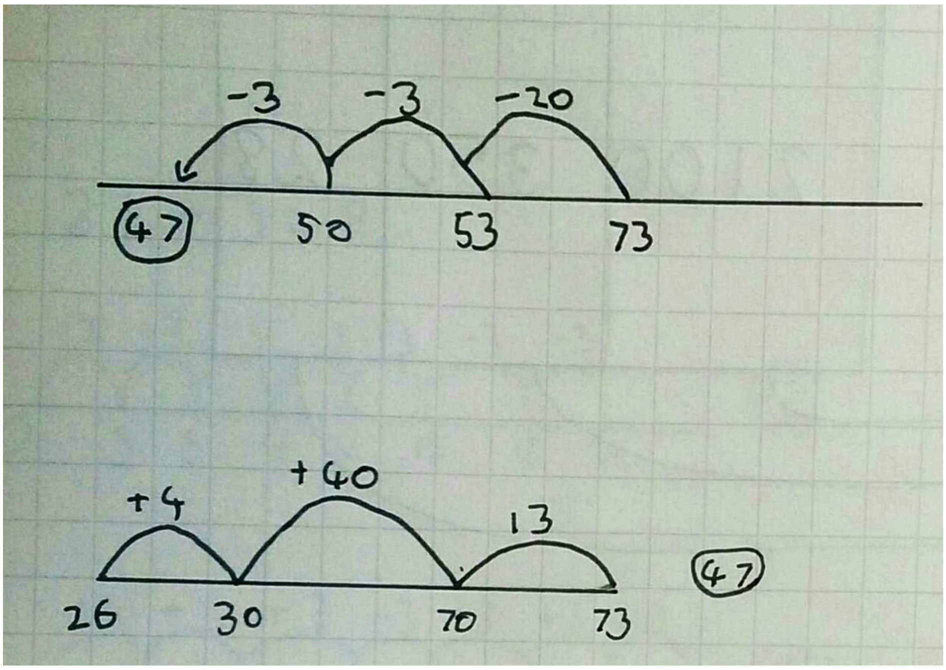


Finding the difference: -



Children should then be encouraged to make jumps of greater than 1.

Example 2: eg 73–26=?

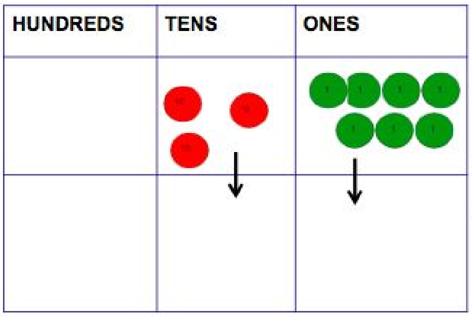


Children should also be encouraged to make sensible decisions of which model to use when calculating on a number line- for example, if calculating 87-13=? Taking away would be more efficient, where as if calculating 87-63=? Finding the difference would be more efficient.

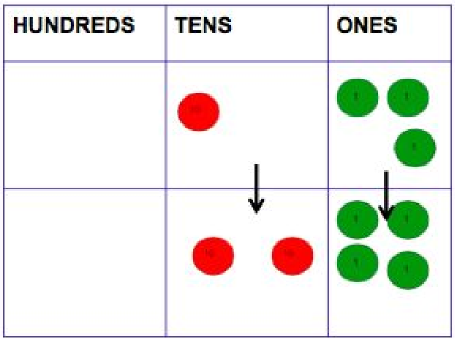
STAGE B:- Formal written method- vertical partitioning with exchange.

To move towards a formal written method, children will need to use their knowledge of subtraction as taking away or finding the difference together with their knowledge of partitioning and exchange. Place value counters are essential when first introducing this concept.

Example 1: eg 98-35=?



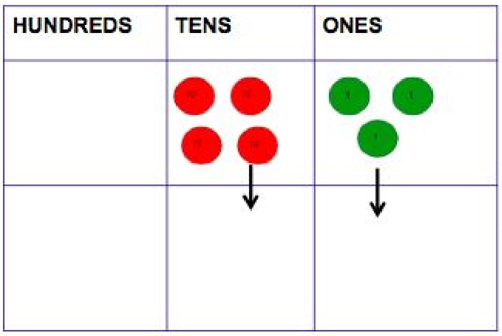
$$\begin{array}{r} 30\ \ 7 \\ -\ 20\ \ 4 \\ \hline \\ \hline \end{array}$$



$$\begin{array}{r} 30\ \ 7 \\ -\ 20\ \ 4 \\ \hline 10\ \ +3 \\ \hline 1\ 3 \end{array}$$

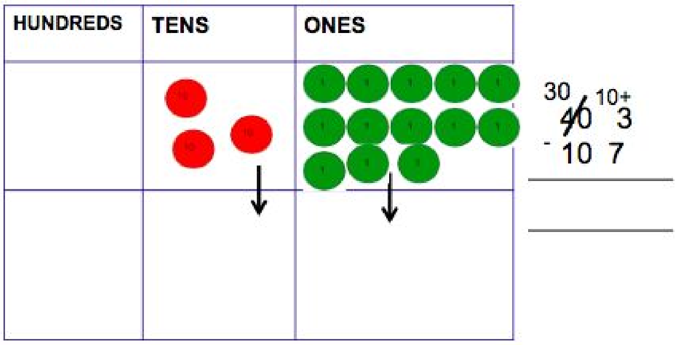
This will then lead to calculations where exchange is needed.

Example 2: eg 43-17=?

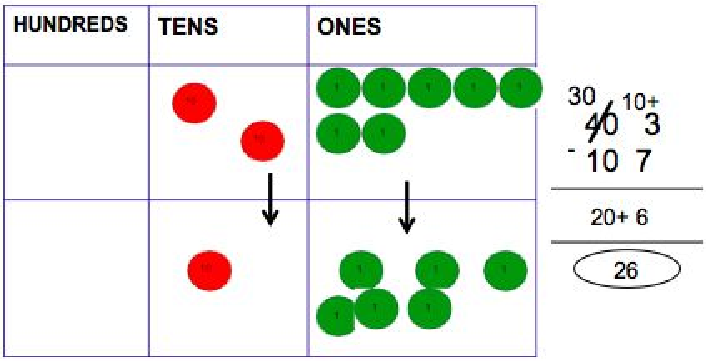


$$\begin{array}{r} 40\ \ 3 \\ -\ 10\ \ 7 \\ \hline \\ \hline \end{array}$$

Children would exchange one ‘10’ for ten ‘1’s’ and record this on the formal layout.



And then complete the calculation.



Children will then progress to being able to carry out the calculation without using a physical place value board **(internalising the representation).**

Example 3: 145- 97=?
The example below includes an example commentary of the conceptual thinking behind the method.

First we canonically partition the numbers and set them out in the formal layout.

$$\begin{array}{rrrr} & 100 & 40 & 5 \\ - & & 90 & 7 \\ \hline & & & \\ \hline \end{array}$$

We need to **exchange** a 10 for ten ‘1’s’ so that we can easily complete the ones-ones partition.

$$\begin{array}{rrrr} & & 30 & 10+ \\ & 100 & \cancel{40} & 5 \\ - & & 90 & 7 \\ \hline & & & \\ \hline \end{array}$$

We then carry out the subtraction of the ones partition

$$\begin{array}{rrrr} & & 30 & 10+ \\ & 100 & \cancel{40} & 5 \\ - & & 90 & 7 \\ \hline & & & 8 \\ \hline \end{array}$$

We need to **exchange** a 100 for ten 1’s and then carry out the subtraction of the tens partition, leading to our final answer of 48

$$\begin{array}{rrrr} & & 100+ & \\ & & 30 & 10+ \\ & \cancel{100} & \cancel{40} & 5 \\ - & & 90 & 7 \\ \hline & & 40+ & 8 \\ \hline & & 48 & \end{array}$$

.

Example 4: eg 456 -147=?

$$\begin{array}{rrrr} & & 40 & 10+ \\ & 400 & \cancel{50} & 6 \\ - & 100 & 40 & 7 \\ \hline & 300+ & 0 & +9 \\ \hline & & 309 & \end{array}$$

When children are conceptually secure with this method, they will be able to further compact the method.

$$\begin{array}{rrrr} & & 4 & {}^{1} \\ & 4 & \cancel{5} & 6 \\ - & 1 & 4 & 7 \\ \hline & 3 & 0 & 9 \\ \hline \end{array}$$

Example 5: eg   4.56-2.63=?
NB before progressing to the compact method with decimals which is shown below, in order to secure the conceptual understating most children will need to initially use the expanded formal method with decimals before progressing to the compact method.

$$\begin{array}{rrrr} & 3 & {}^{1} & \\ & \cancel{4}. & 5 & 6 \\ - & 2. & 6 & 3 \\ \hline & 1. & 9 & 3 \\ \hline \end{array}$$

Progression in Calculations-

Multiplicative Reasoning

(Multiplication and Division)

Multiplicative Reasoning- Multiplication

Multiplication is the process of repeated addition. The inverse of multiplication is division.

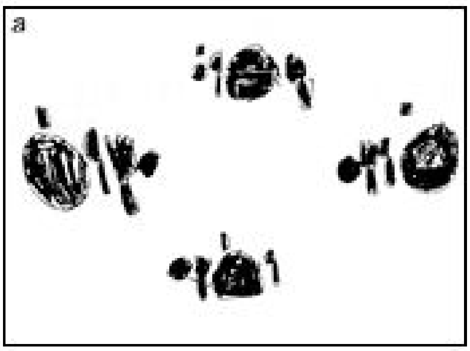
Key Vocabulary
multiply, times, product, lots of, groups of, multiples, arrays

Early Learning- Multiplication

Children will Count repeated groups of the same size

2 + 2 + 2 = 6   (2 x 3 = 6)

They work on practical problem solving activities involving equal sets or groups.



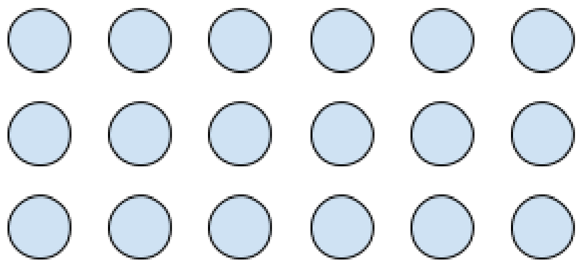
Mental Strategies- Multiplication

Progression	Count on from and back in ones, twos, fives and tens. Derive doubles of numbers to 20 and doubles of multiples of 10 to 100. Recognise multiples of 2, 5 and 10.
	Derive and recall multiplication facts for 2, 3, 4, 5, 6, 10 times tables. Identify doubles of 2-digits numbers. Derive doubles of multiples of 10 and 100. Recognise multiples of 2, 3, 4, 5, 6 and 10 . Derive and recall all multiplication facts up to 12-times. Recognise multiples up to 12.
	Recognise square numbers up to 12x12. Multiply a 2-digit number by a single digit. Use place value and multiplication facts to derive decimals: ie 0.6x8 =4.8 Identify prime numbers less than 100.

Doubling

Progression	Double all numbers to 20. Double multiples of 10 to 100. Double multiplies of 5 to 100.
	Double any 2-digit number to 100. Double multiples of 10 and 100. And all corresponding halves of above. Multiply by 4, double and double again. Multiply by 8, double three times. Multiply by 5, multiply by 10 and half. Multiply by 20, multiply by 10 and double.
	Multiply by 50, multiply by 100 and half. Multiply by 25, times 100 and half and half again. Double decimals with units, tenths and hundredths.

Written Strategies- Multiplication

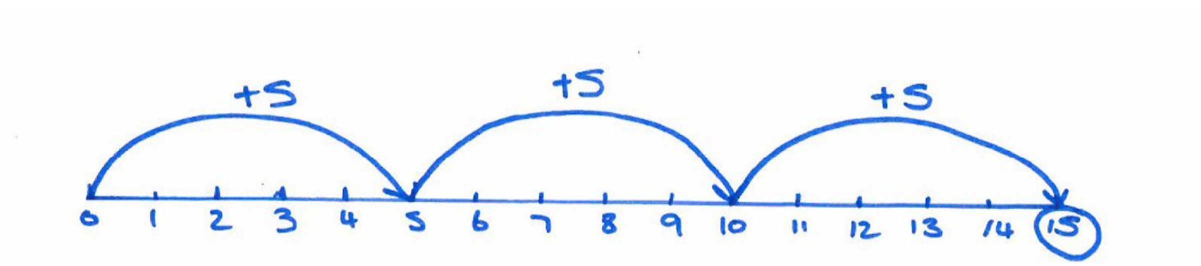
Key Concepts
Arrays Arrays underpin all teaching of multiplication. For example, 3 x 6 can be thought of as 3 lots of 6 or 6 lots of 3. 

Key Concepts
Repeated Addition 5 x 3 = 5 + 5 + 5 = 15 This links to arrays

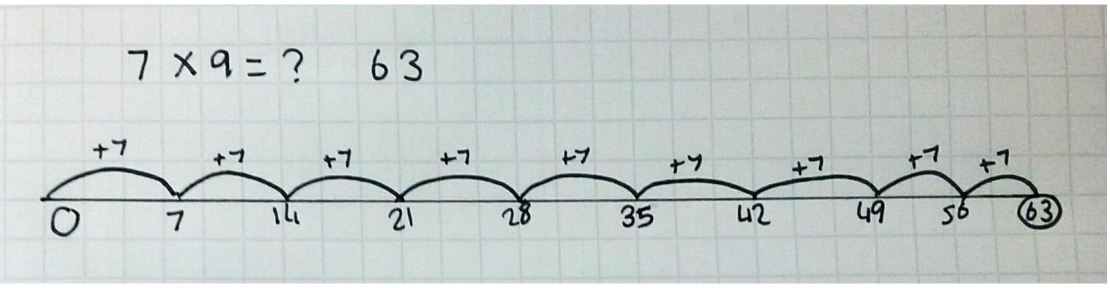
STAGE A:- Number line- repeated addition (linked to the array)

Repeated addition can be shown easily on a number line

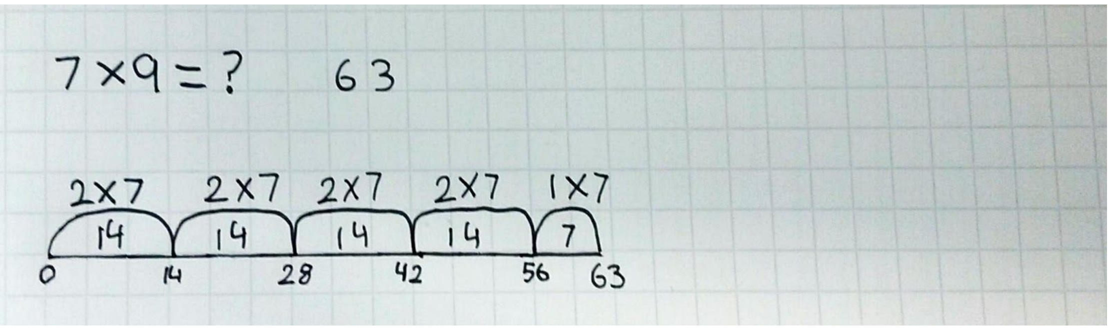
Example 1: 5 x 3= 15



Example 2: 7 x 9=63



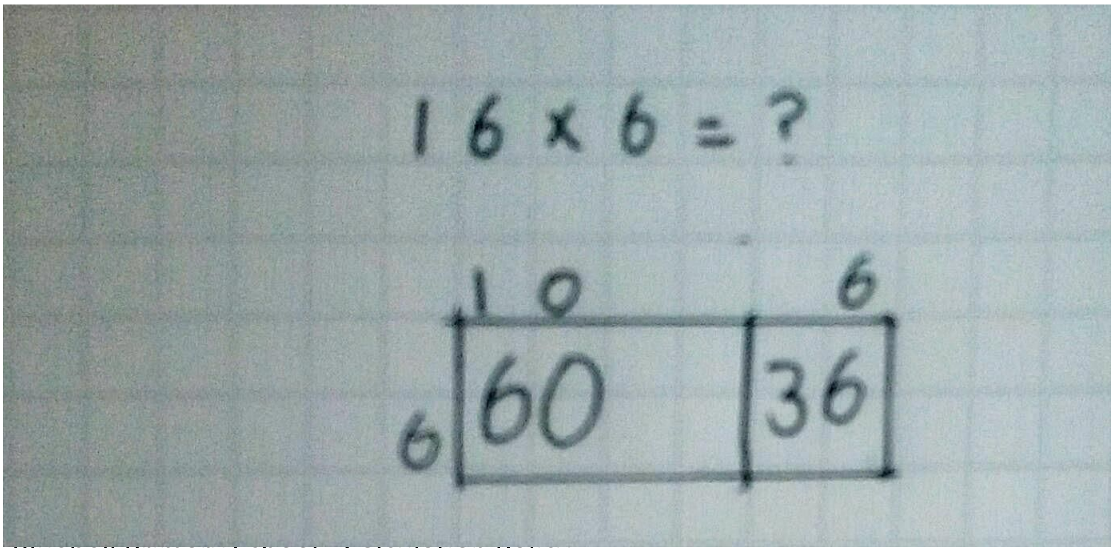
Children can also use their known facts to help them calculate more efficiently using repeated addition.



STAGE B:- Informal array written method.

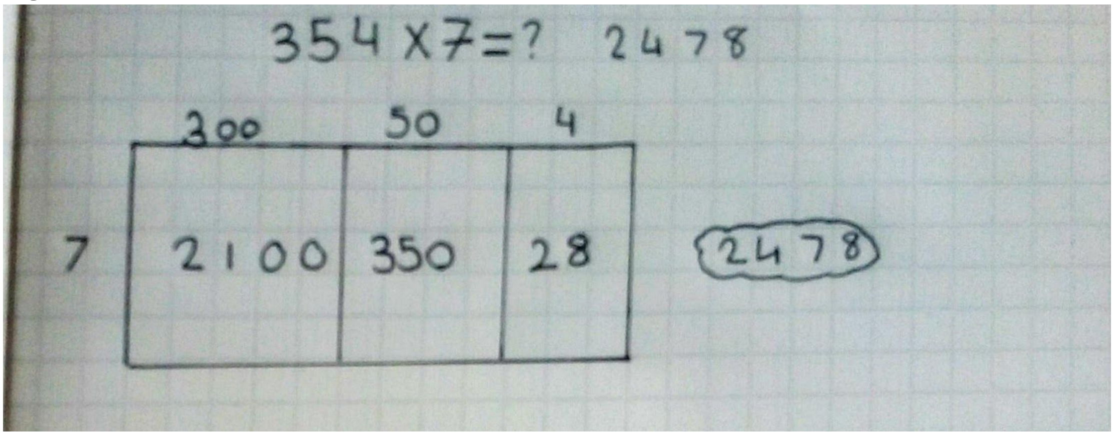
Children can initially make large arrays out of cubes and other objects, and partition these. They will then move onto more abstract representations.

Example 1 eg 14 x 6 = ?

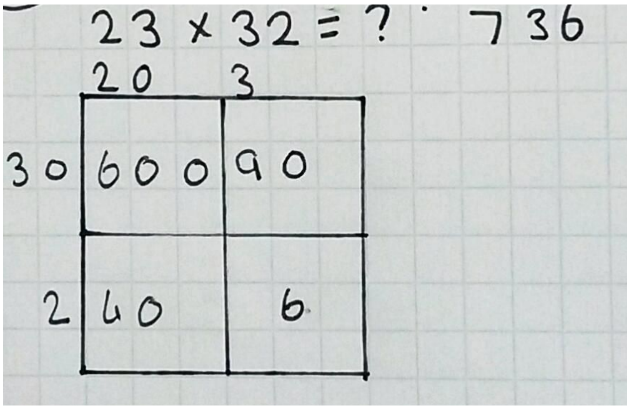


Children then progress to using the informal array method for larger numbers.

Example 3: eg 354 x 7 = ?



Example 4: eg 23 x 32 = ?



STAGE C:- The formal written method (array linked).

Once children are conceptually secure with the informal array written method, this can then be extended into the formal written method for multiplication. Children may be ready to calculate HTO x O =? calculations using the formal written method, but still need to use the informal array method for TOxO=?

Key Concept

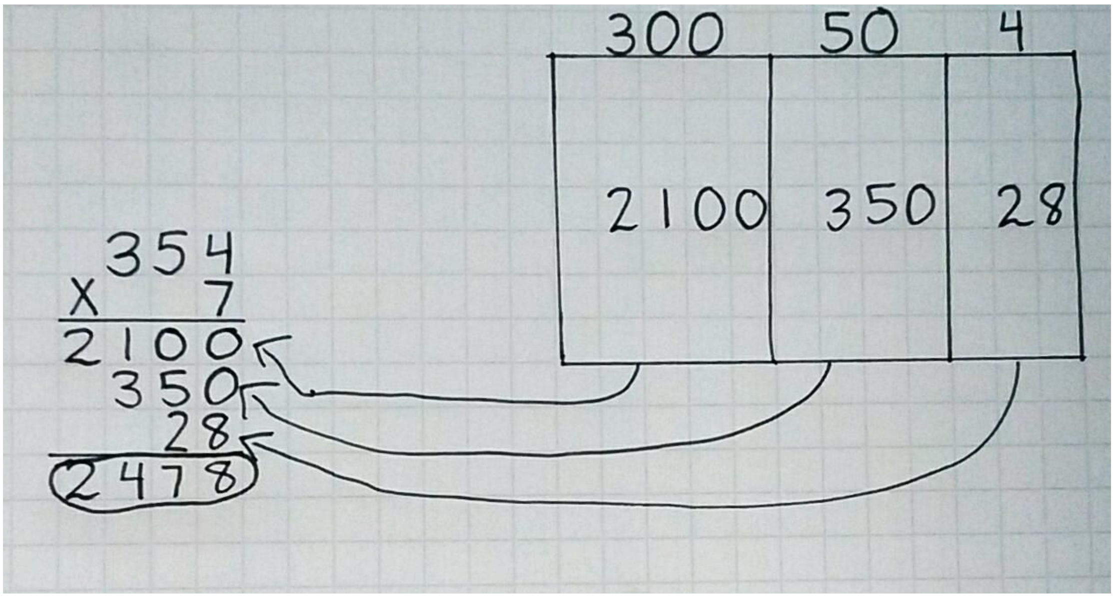
Keeping the link to arrays.
Links between the formal written method and arrays should be maintained.

The formal written method is a more efficient way of recording the different parts of an array.

Children should, at first, be encouraged to record using **both** the informal array written method and the formal written method, so that the link between the array and the formal written method is secure. Question structures such as 'What's the same, what's different'? and 'What do you notice?' should be used to help children make comparisons and see the links between the methods.

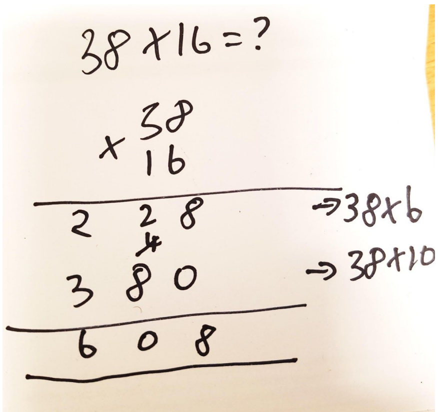
This should be repeated with each increase in difficulty (e.g. when looking at HTO x O [e.g. 342 x 5=?] , again when looking at TO x TO [23 x 32 =?] and then again when looking at O.t x O.t [e.g. 2.3 x 4.3=?])

Example 1: eg 346x 7=?

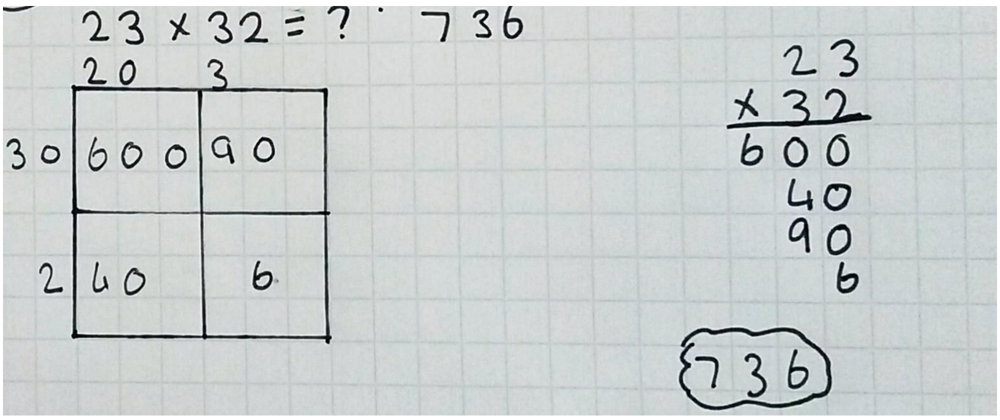


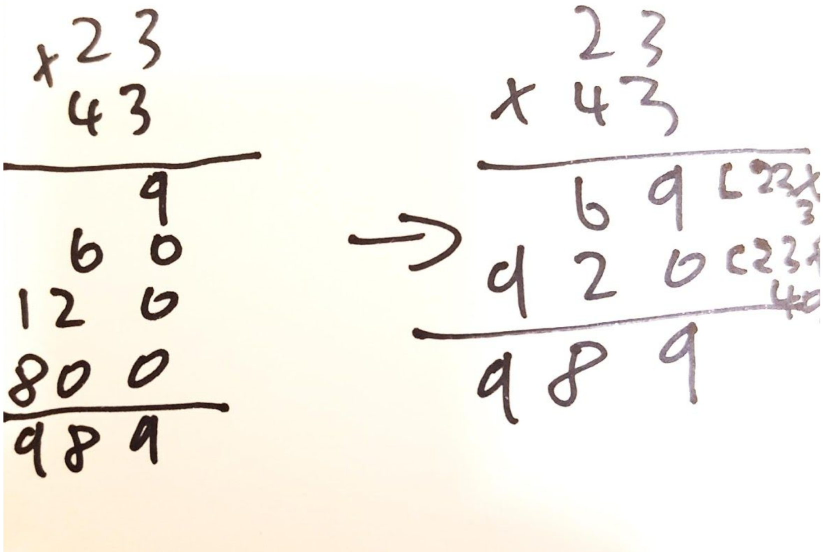
Example 2: eg 23 x 32=?

Children can then make this method even more efficient by using their mental calculation

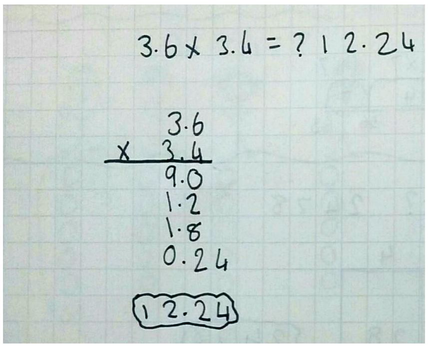


Example 3: eg 23 x 43=?





Example 4: eg   3.6 x 3.4=?



Multiplicative Reasoning- Division

Division is when you **group** or **share** objects or amounts. It is the inverse of multiplication

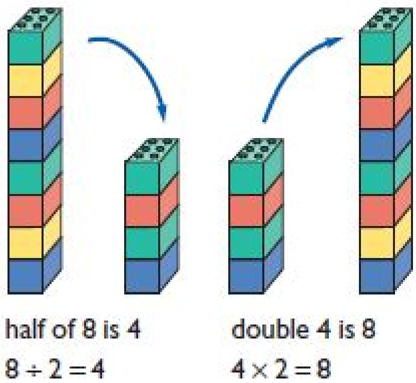
Key Vocabulary
grouping, sharing, remainder,

Early Learning- Division.

Understanding division as sharing
6 cakes are shared between 2 people. How many cakes do they have each?

Children use a range of representations to represent division as both grouping and sharing.

Children will also become familiar with the language and action of halving (as the inverse of doubling).



Children use division in day to day life- they may share items between friends, double or half quantities etc.

Mental Strategies- Division

Multiplication and Division Facts to 10x

Children should develop a rapid recall of division facts within their 1-10x tables, as this supports many different calculations, including the written strategies for multiplication contained within this policy.

Progression	Derive and recall division facts for the 2,5 and 10 times table.
	Derive and recall division facts for the 2,3,5 and 10 times table.
	Derive and recall division facts for the 2,3,4,5, 6 and 10 times table.
	Derive and recall division facts for the 2-10 times table.
	Derive and develop rapid recall of division facts for the 2-10 times table.
	Derive division facts for pairs of multiples of 10.
	Derive division facts for decimals (e.g. 4.8 ÷ 6=?). Recall the square roots of all square numbers to 10^2.

Halving

Children should recognise halving as the inverse of doubling and be able to rapidly calculate halves of numbers.

Progression	Find the corresponding halves for all doubles of numbers to 10 (e.g. half 18)
	Find the corresponding halves for all doubles of numbers to 20 (e.g. half 18), multiples of 10 to 50 (e.g. half 80) and multiples of 5 to 50 (e.g. half 30)
	Find the corresponding halves for all doubles of multiples of 10 to 100 (e.g. half 180) and multiples of 5 to 100 (e.g. half 70)
	Find the corresponding halves for all doubles of any two digit number (e.g. half 84) and any multiple of 10 or 100 (e.g half 680)
	Divide by 4 by halving twice and 8 by halving again. Multiply by 50 by multiplying by 100 and halving (e.g. 8 x 50= 8 x100 divided by 2)
	Find the corresponding halves for all doubles of decimals with units and tenths (e.g. half 8.4)
	Divide by 25 by dividing by 100 then multiplying by 4 Divide by 50 by dividing by 100 then doubling.

Dividing by Multiples of 10.

Being able to divide by 10 and multiples of 10 depends on an understanding of place value and knowledge of multiplication and division facts. This ability is fundamental to being able to multiply and divide larger numbers without resorting to a written method.

Progressi on	Recall division facts for the 10 times table.
	Divide numbers to 1000 by 10 progressing to 100.
	Divide whole numbers and decimals by 10, 100 or 1000. Divide a multiple of 10 by a single digit number (whole number answers)
	Divide multiples of 100 by a multiple of 10 or 100 (whole number answers) e.g. 600÷20 Divide by 25 or 50.

Fractions, Decimals and Percentages

Children need an understanding of how fractions, decimals and percentages relate to each other and how they are related to division. For example, if they know that ½ 0.5 and 50% are all ways of representing the same part of a whole, then they can see that the calculations:

Half of 40
40 ÷2
½ of 40
40 × ½
40 × 0.5
0.5 × 40
50% of 40

are different versions of the same calculation. Sometimes it might be easier to work with fractions, sometimes with decimals and sometimes with percentages.
There are strong links between this section and the earlier section 'Dividing by multiples of 10'.

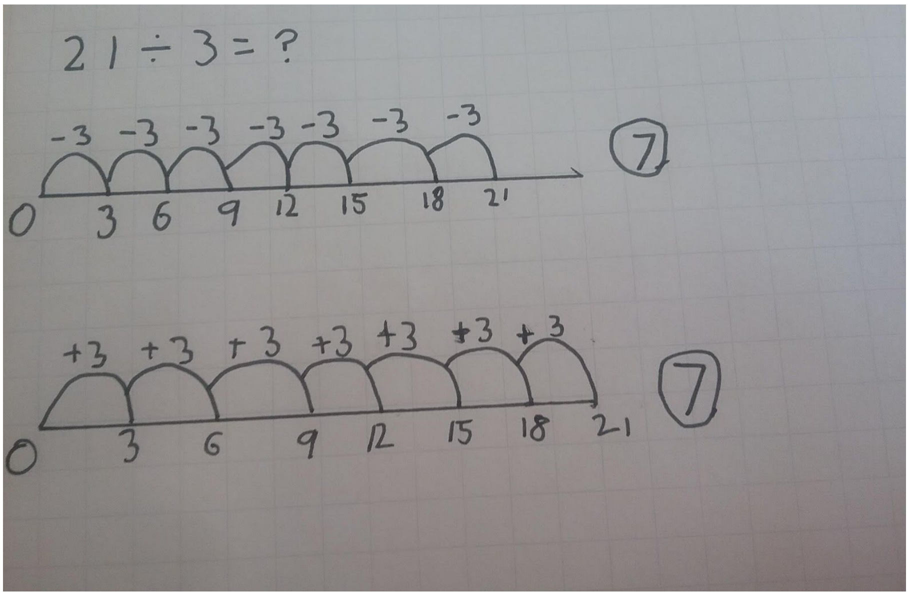
Progression	Find half of any even number to 40 or a multiple of 10 to 100- e.g. half of 80.
	Find half of any multiple of 10 up to 200, e.g. half 170 Mentally find , , , of numbers in the 2,3,4,5 and 10 times table.
	Divide numbers to 1000 by 10 progressing to 100.
	Recall percentage equivalents to , , , tenths and hundredths. Mentally find fractions of whole number or quantities using division. Mentally find 50%, 25% or 10% of whole number of quantities- e.g. 25% of 20kg, 10% of £80.
	Recall equivalent fractions, decimals and percentages for hundredths. Mentally find half of decimals with units and tenths (e.g. half of 3.2) Find 10% or multiples of 10% of whole numbers and quantities- e.g. 30% of 50ml.

Written Methods- Division

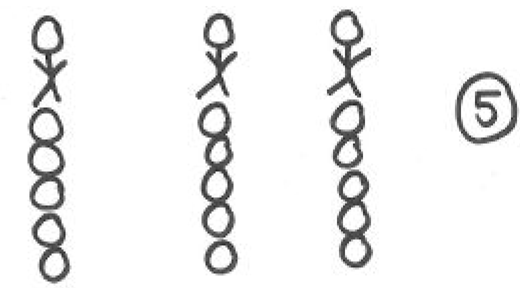
STAGE A:- Early Exploration

Whilst most exploration of division in the earlier stages of a child's mathematical education should be through manipulating practical apparatus, the model of division as grouping can be recorded on a number line through either addition or subtraction.

Example 1: eg 21 ÷ 3 =?



e.g 15 ÷ 3=?

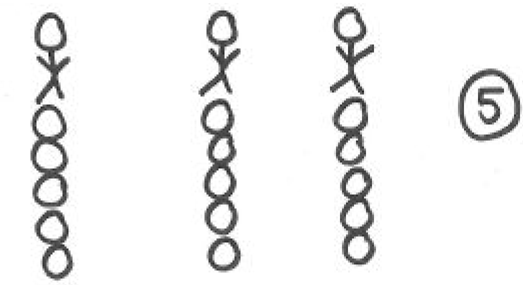


STAGE B Sharing- informal written method (conceptually based)

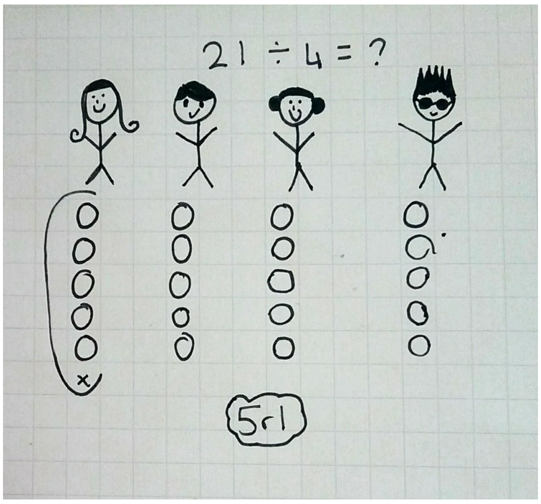
Key Concepts
Inverse Children should be familiar with the multiplication table of the divisor they are working with. For example, with 15 ÷ 3 they should be confident with thee 3 times table

Children can build on their early learning and use the structure of division as sharing to calculate using division questions. Initially using counters and then recording on paper.

Example 2: eg 15 ÷ 3 =?

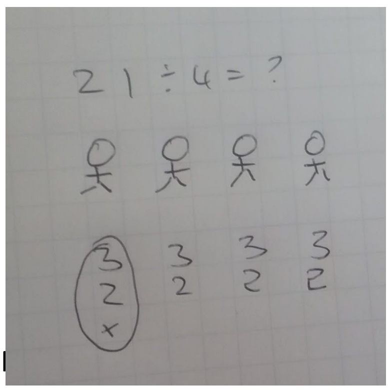


Example 2: eg 21 ÷ 4 =
Children can also work this way with remainders,, which should be introduced at an early stage.



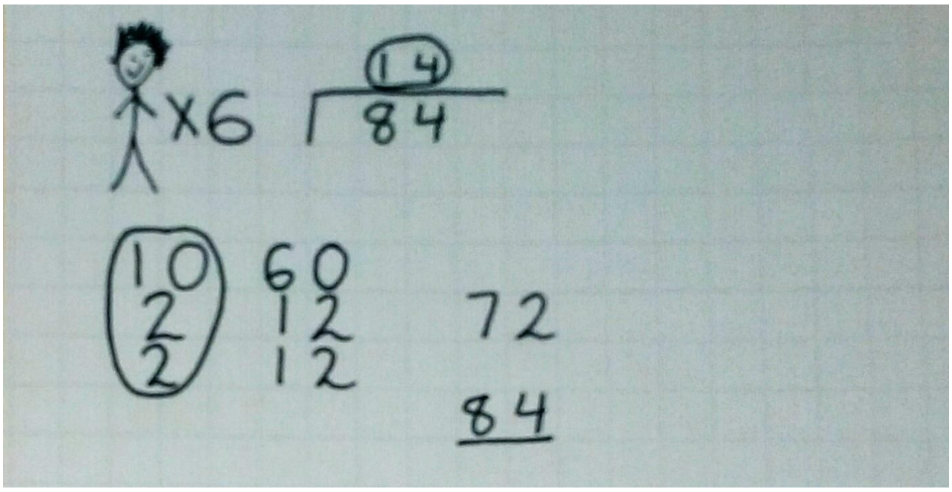
This can be developed into a more abstract form

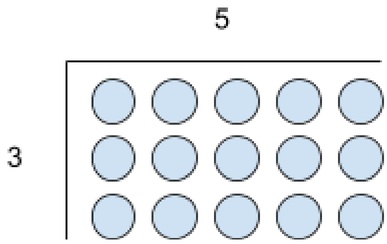
(To share 21 between 4 you are giving groups of 3 and 2 to each person, the x represents your remainder)
And the formal algorithm notation can be introduced



Example 3: eg 84 ÷ 6 =?

(To share 84 between 6 you are giving groups of 10, 2 and 2 to 6 people)



Key Concepts
Division as an array The formal layout for division can be thought of as an array. E.g. 12 ÷ 4 = 

STAGE C:- Formal written method- Short division (Arrays)- with place value counters_ (previously known as the bus stop method)

Key Concept
Order When teaching this method, it is vital that children are taught that they are sharing the dividend by the divisor (e.g. 989 shared between 8). They should be taught that they are using their partitioning and exchange skills to share 9 'hundreds' between 8 etc… They should **NEVER** be taught that they see 'how many 8's go into 9' (ie.. divisor by the dividend)- this is **not** conceptually based in any way and does not enable the children to make connections and spot errors in their working.

The model of division as an array can be used to develop an efficient formal written method, which is sometimes referred to as the 'bus stop' method.

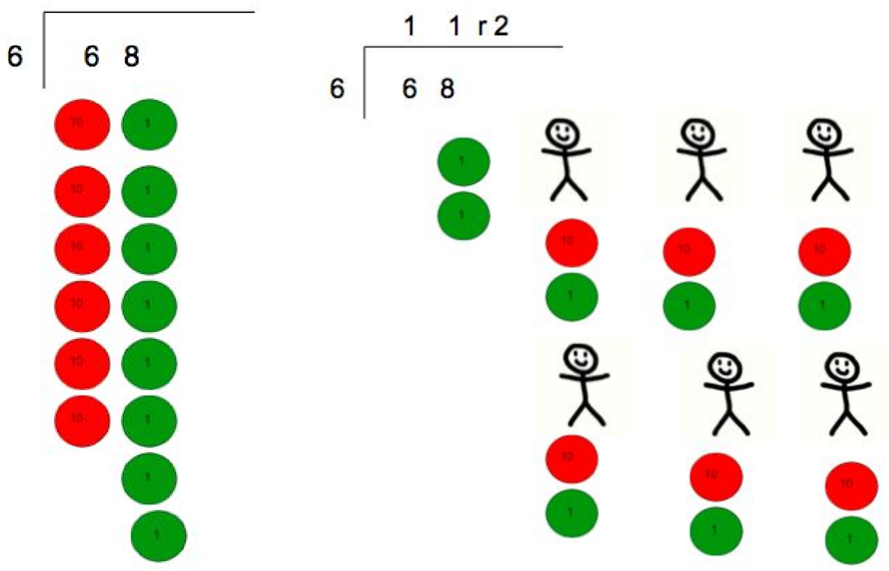
The model of place value counters can be used to share the **dividend** between the **divisor**- this links directly to the informal written sharing method of division.

Children share each 'place' (e.g. hundreds, tens, ones) individually. They use their knowledge and competence in exchange to deal with any remainders.

Initially, children will work with questions where there is no need to exchange. They represent the dividend using place value counters and share these between the divisor.

NB- place value counters are shown below as an illustration- children should not draw theses in their books!

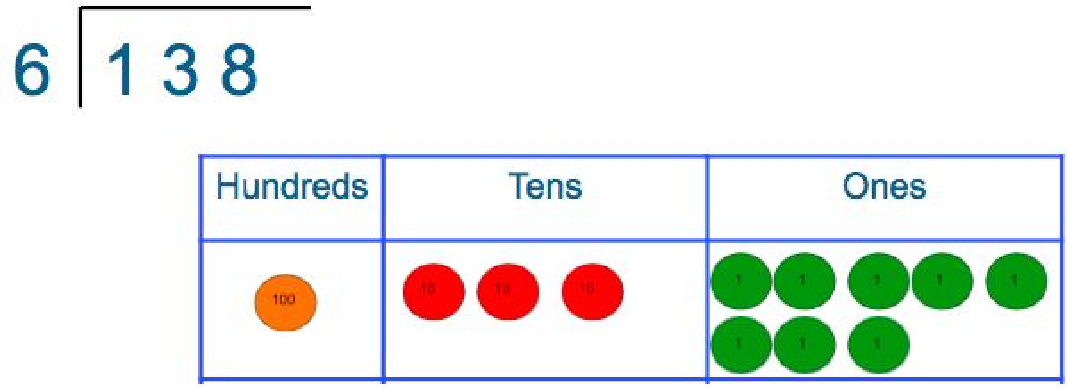
Example 1: 68 ÷ 6 =?



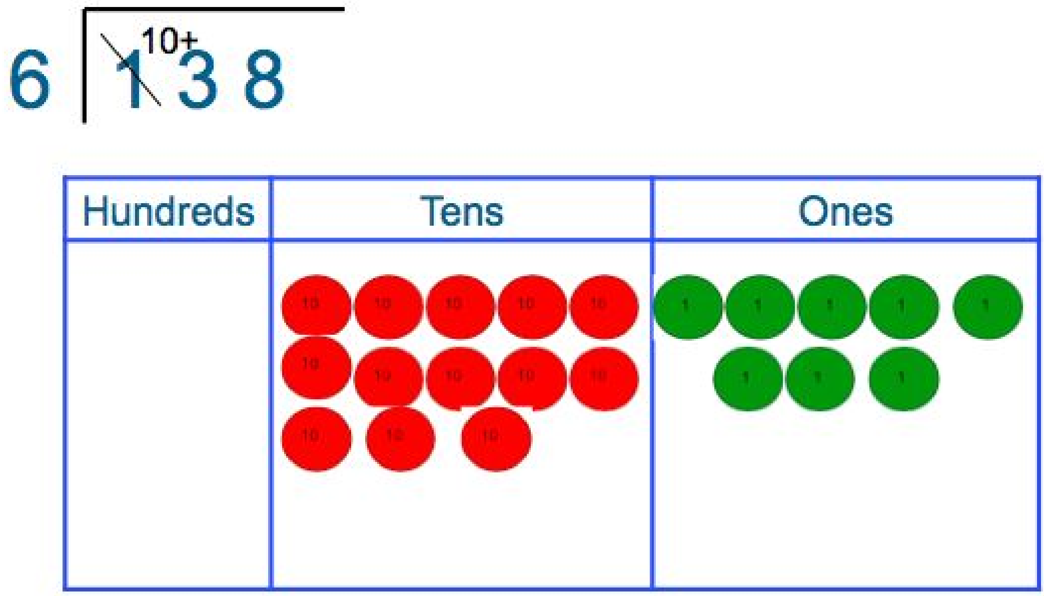
Then children will begin to be able to use their knowledge of exchange to solve division questions where exchange is required.
Place value mats are particularly helpful at this stage.

NB- the following example takes up a lot of printed space- please remember that children will be doing this using physical place value counters.

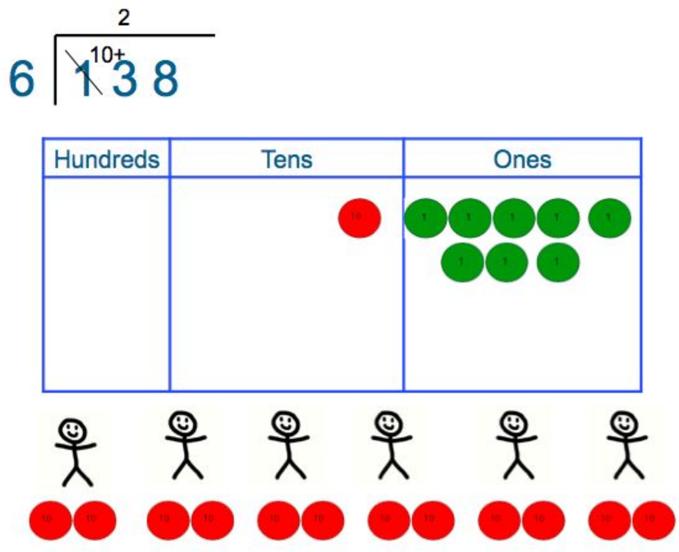
Example 2: 138 ÷ 6 =?



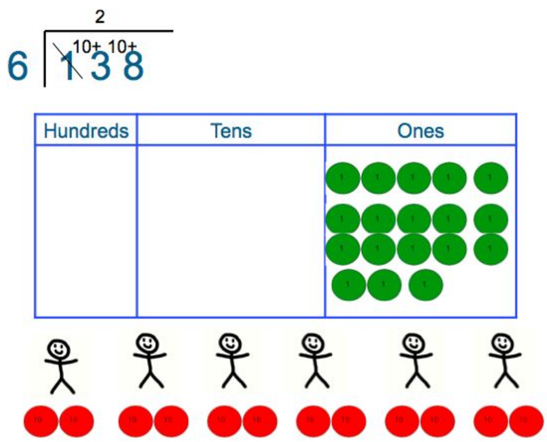
Children will realise they cannot easily share a whole 100 (one 100 counter) between 6. But they can **exchange** their 100 for ten 10's. They record this on their formal algorithm.



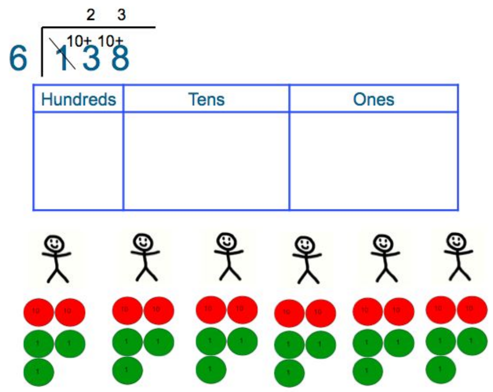
They can then share their 'tens' between 6. They record their answer in on the formal algorithm above the tens place. Ensure the children are secure with the fact that a 2 in the tens place (as is the case here) represents 20.



They then use their knowledge of exchange to exchange the one remaining ‘10’ for ten ‘1’s- again recording this on their standard algorithm.



They can then share their ‘ones’ between 6, again recording the results on their standard algorithm.



If there are any ‘ones’ left over after sharing between the divisor, then this is the remainder.

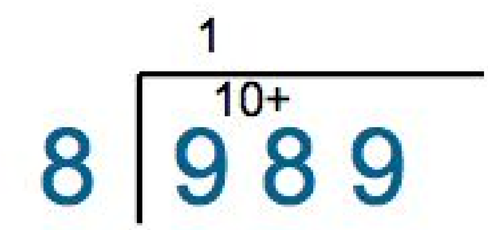
Children need to be able to record the remainder as an integer, decimal and fraction.

Once children are secure with the conceptual base (using place value counters) they then progress to doing this using a purely written method. A stepped example, along with commentary is provided below. **This example is one of the most complex levels of calculation that children would be expected to carry out using this method of short division.**

Example 3: 989 ÷ 8 =?

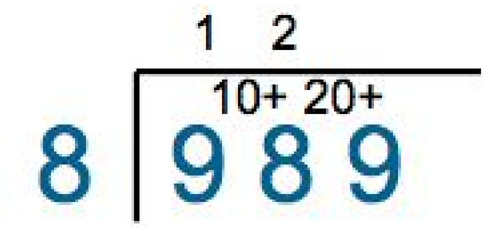
I have nine '100's'. I can share one '100' to each of the 8 'people'. I'm going to record this above the hundreds place.

This leaves me with one '100' left over. I can **exchange** this for ten '10's.



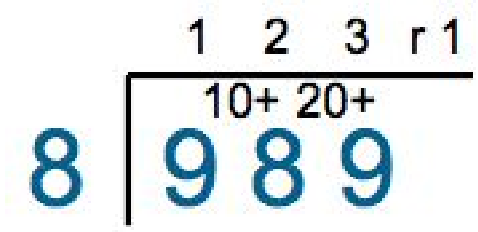
I know have eighteen '10's'. I can share two '10s' to each of the 8 'people'. I'm going to record this above the tens place.

This leaves me with two '10's' left over. I can **exchange** this for twenty '1's'

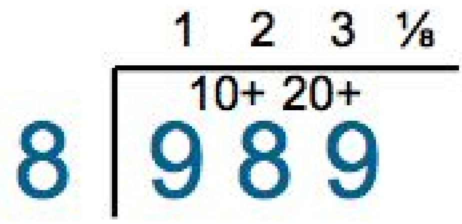


I know have twenty-nine '1's'. I know that 3 x 9=28 so I can share 3 of these to each of my 8 'people' . I'm going to record this above the ones place.

This leaves me with 1 left over, so I have a remainder of 1.



or



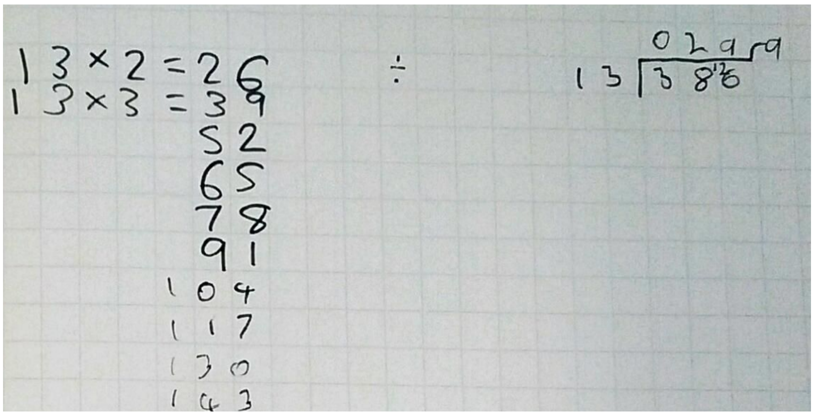
Once children are very secure with this method, they may decide they can miss recording the ones digits of any exchange, leading to a final method that could look like this.

STAGE D:- Long Division- Formal written method- with ‘golden nuggets’

The formal algorithm for long division which we teach is the same as short division. However children will benefit from recording ‘golden nuggets’- the key multiplication facts that they can easily work out, to help them with their calculations.

This is a variation of the method set out in the appendices of the national curriculum and will gain method marks in the national curriculum assessments.

Example 1: eg 386 ÷ 13=?



What If?

What if pupils prefer a different method?

Tell the children that within your school we are taught how to use these methods to help them understand the calculation and they need to try and use these methods within school.

What if pupils join the school with a different method?

Discuss how their previous methods worked and show them how we complete the methods in our school. Explain that everyone in the school can choose from the methods we teach and give them the chance to practice and understand how they work.

What if pupils 'can't' do the method?

If pupils 'can't' do the methods they are telling you that they do not have the conceptual understanding of the calculation and need to go back a step or even more. This will ensure they have the place value understanding to help them to complete the method.

What if parents don't like the methods adopted?

Explain to the parents that we have chosen these methods as we feel this help these help the children to understand why they have to complete certain steps to gain the answer. Offer the parents the chance to show them how the methods work and encourage them to use the same ones at home.

What if parents teach another method?

If parents teach the children a different method, tell the children that they can use those methods at home if they wish but in school they have to use the methods that we teach.

What if all teaching staff are not able to support the methods?

If you are not confident in supporting the methods to the children then seek help yourself. Ask people within your year group or your Maths Co-ordinator and they would be willing to explain to you how you can help support the children to gain a better understanding.